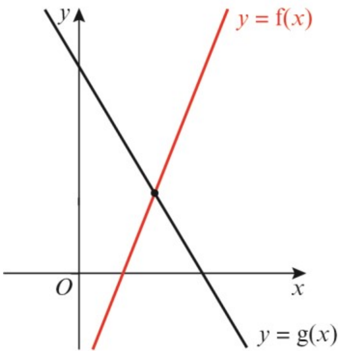
Exercise 3F

1 a $2y + 3x = 6$ **(1)**
$x - y = 5$ **(2)**
Multiply equation (**2**) by 2:
$2x - 2y = 10$ **(3)**
Add equations (**1**) and (**3**):
$5x = 16$
$x = \frac{16}{5}, y = -\frac{9}{5}$
The solution is $P\left(\frac{16}{5}, -\frac{9}{5}\right)$.

b $2y + 3x > x - y$ when the line L_1 is above the line L_2:
$x < \frac{16}{5}$

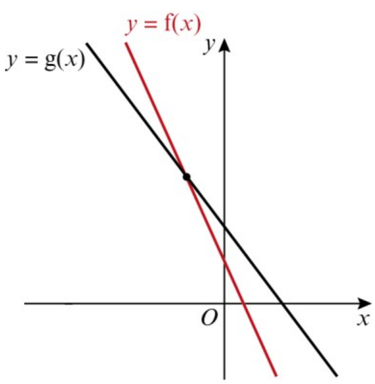
2 a i



ii $3x - 7 = 13 - 2x$
$5x = 20$
$x = 4, y = 5$
The lines intersect at (4, 5).

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $x \leq 4$

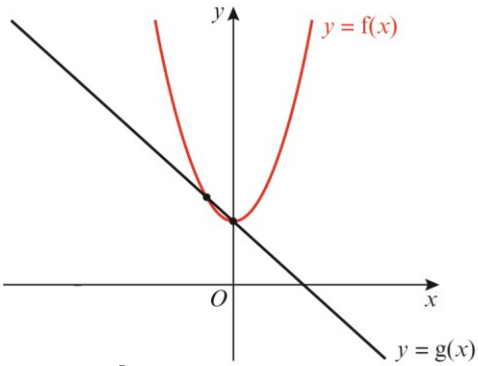
b i



2 b ii $8 - 5x = 14 - 3x$
$-2x = 6$
$x = -3, y = 23$
The lines intersect at (−3, 23).

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $x \geq -3$

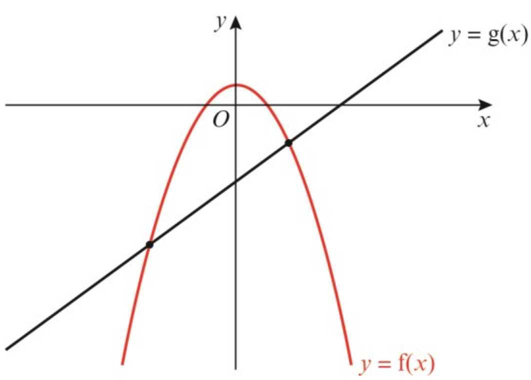
c i



ii $x^2 + 5 = 5 - 2x$
$x^2 + 2x = 0$
$x(x + 2) = 0$
$x = 0$ or $x = -2$
When $x = 0, y = 5$
When $x = -2, y = 9$
The lines intersect at (0, 5) and (−2, 9).

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $-2 \leq x \leq 0$

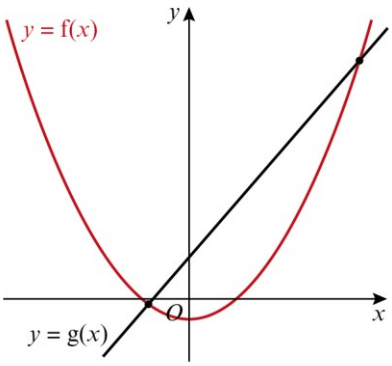
d i



2 d ii $3 - x^2 = 2x - 12$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$x = -5$ or $x = 3$
When $x = -5$, $y = -22$
When $x = 3$, $y = -6$
The lines intersect at $(-5, -22)$ and $(3, -6)$.

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $x \leq -5$ or $x \geq 3$

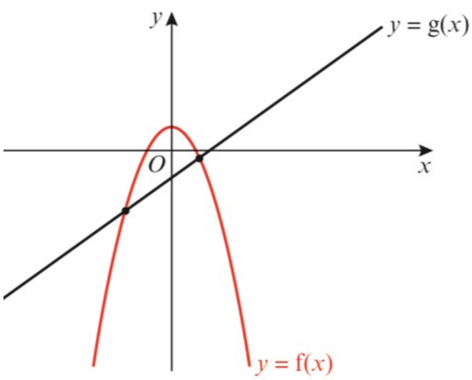
e i



ii $x^2 - 5 = 7x + 13$
$x^2 - 7x - 18 = 0$
$(x - 9)(x + 2) = 0$
$x = 9$ or $x = -2$
When $x = 9$, $y = 76$
When $x = -2$, $y = -1$
The lines intersect at $(-2, -1)$ and $(9, 76)$.

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $-2 \leq x \leq 9$

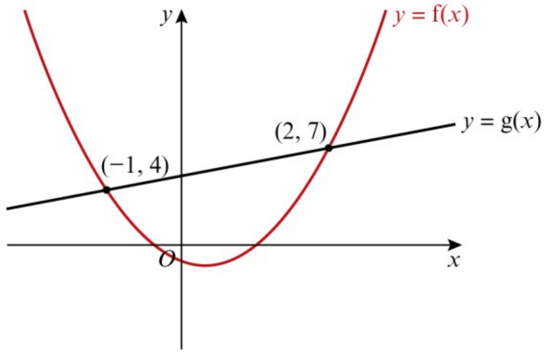
f i



f ii $7 - x^2 = 2x - 8$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$x = -5$ or $x = 3$
When $x = -5$, $y = -18$
When $x = 3$, $y = -2$
The lines intersect at $(-5, -18)$ and $(3, -2)$

iii $f(x) \leq g(x)$ when $f(x)$ is below $g(x)$, so when $x \leq -5$ or $x \geq 3$

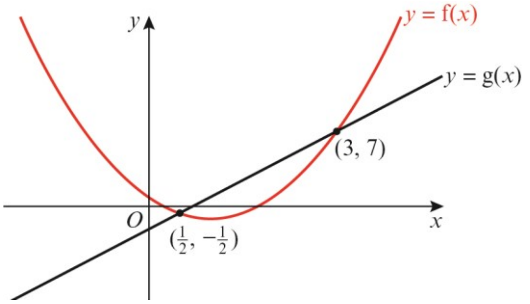
3 a $3x^2 - 2x - 1 = x + 5$
$3x^2 - 3x - 6 = 0$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2$, $x = -1$
The points of intersection are $(2, 7)$ and $(-1, 4)$.



So the required values are $-1 < x < 2$.

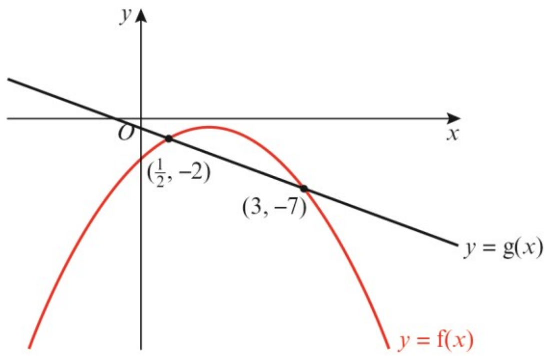
b $2x^2 - 4x + 1 = 3x - 2$
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or $x = 3$
The points of intersection are $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and $(3, 7)$.

3 b



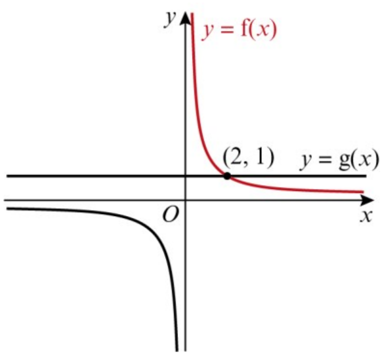
So the required values are $\frac{1}{2} < x < 3$.

c $5x - 2x^2 - 4 = -2x - 1$
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or $x = 3$
The points of intersection are
$(\frac{1}{2}, -2)$ and $(3, -7)$.



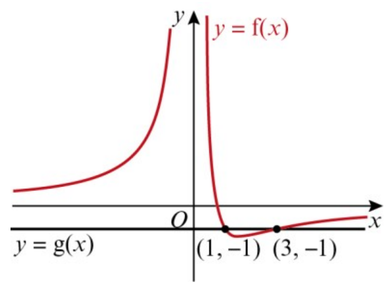
So the required values are
$x < \frac{1}{2}$ or $x > 3$.

d $\frac{2}{x} = 1 \Rightarrow x = 2$
Point of intersection is (2, 1)



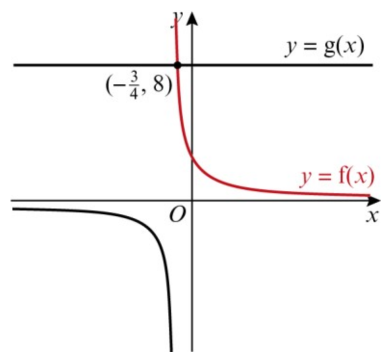
d So the required values are
$x < 0$ or $x > 2$.

e $\frac{3}{x^2} - \frac{4}{x} = -1$
Multiply both sides by x^2:
$3 - 4x = -x^2$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1$ or $x = 3$
Points of intersection are
(1, −1) and (3, −1)



So the required values are $1 < x < 3$.

f $\frac{2}{x+1} = 8$
$2 = 8(x + 1)$
$8x + 6 = 0$
$x = -\frac{3}{4}$
Point of intersection is $(-\frac{3}{4}, 8)$



So the required values are
$x < -1$ or $x > -\frac{3}{4}$.

Challenge

a $x^2 - 4x - 12 = 6 + 5x - x^2$
$2x^2 - 9x - 18 = 0$
$(2x + 3)(x - 6) = 0$
$x = -\frac{3}{2}$ or $x = 6$
The points of intersection are
$(-\frac{3}{2}, -\frac{15}{4})$ and $(6, 0)$.

b The required values are $-\frac{3}{2} < x < 6$
$\{x: -\frac{3}{2} < x < 6\}$